Abstraction (Q*-irrelevant)

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ϕ : Q^*-irrelevant : $\forall s^{(1)}, s^{(2)} : \phi(s^{(1)}) = \phi(s^{(2)})$, $\forall a \in A$, $Q^*_M(s^{(1)}, a) = Q^*_M(s^{(2)}, a)$.

$M_\phi = (S_\phi, A, P_\phi, R_\phi, \gamma)$. $\forall x, a, x'$,

$R_\phi(x,a) = R(s,a)$, for arbitrarily chosen $s \in \phi^{-1}(x)$

$P_\phi(x'|x,a) = P(x'|s,a) := \sum_{s' \in \phi^{-1}(x')} P(s'|s,a)$. for arbitrary $s \in \phi^{-1}(x)$

Consistent

w.t.s. $[Q^*_{M_\phi}]_M = Q^*_M$

Define $[Q^*_M]_\phi(x,a) := Q^*_M(s,a)$, $\underline{\forall s \in \phi^{-1}(x)}$. will show : $[Q^*_M]_\phi = Q^*_{M_\phi}$.

suffices to show: $\mathcal{T}_{M_\phi}[Q^*_M]_\phi = [Q^*_M]_\phi$. $\Longleftrightarrow$ $\forall (x,a)$ $[LHS](x,a) = [RHS](x,a)$

$[LHS](x,a) = R_\phi(x,a) + \gamma \langle P_\phi(x,a), [V^*_M]_\phi \rangle$

representive state

$= R(s,a) + \gamma \langle [P(x'|s,a)]_{x'}, [V^*_M]_\phi \rangle$

$= R(s,a) + \gamma \langle P(s,a), V^*_M \rangle$

$= Q^*_M(s,a) \overset{?}{=} [Q^*_M]_\phi(x,a)$.

$\langle P(s,a), V^*_M \rangle$

$= \langle P(s,a), \Phi^T [V^*_M]_\phi \rangle$.

$= \langle \underbrace{\Phi P(s,a)}_{[P(x'|s,a)]_{x'}}, [V^*_M]_\phi \rangle$.