▷ bisimulation ⇒ Q^* - irrelevance.

$M_\phi = (S_\phi, A, P_\phi, R_\phi, \gamma)$

ϕ : $\forall s^{(1)}, s^{(2)}$ s.t. $\phi(s^{(1)}) = \phi(s^{(2)})$

$R(s^{(1)}, a) = R(s^{(2)}, a) = R_\phi(x, a)$

$P(x' | s^{(1)}, a) = P(x' | s^{(2)}, a) = P_\phi(x' | x, a)$.

▷ $[Q^*_{M_\phi}]_M(s,a) = Q^*_{M_\phi}(\phi(s), a)$.

lifting

Claim: $[Q^*_{M_\phi}]_M = Q^*_M$.

Def: $\Phi(x, s) = \mathbb{I}[\phi(s) = x]$.

Φ = S_ϕ × S matrix

$[P(x' | s,a)]_{x' \in S_\phi} = \Phi \times P(\cdot | s, a)$

$\Rightarrow \quad \Phi \times P(\cdot \mid s^{(1)}, a) = \Phi \times P(\cdot \mid s^{(2)}, a)$

$$[V^*_{M_\phi}]_M = \Phi^T V^*_{M_\phi}.$$

$$\begin{bmatrix} \vdots & \\ & \ddots \end{bmatrix} \begin{bmatrix} V^*_{M_\phi} \end{bmatrix}$$

$$\left(\mathcal{T}_M [Q^*_{M_\phi}]_M\right)(s,a) \overset{?}{=} [Q^*_{M_\phi}]_M$$

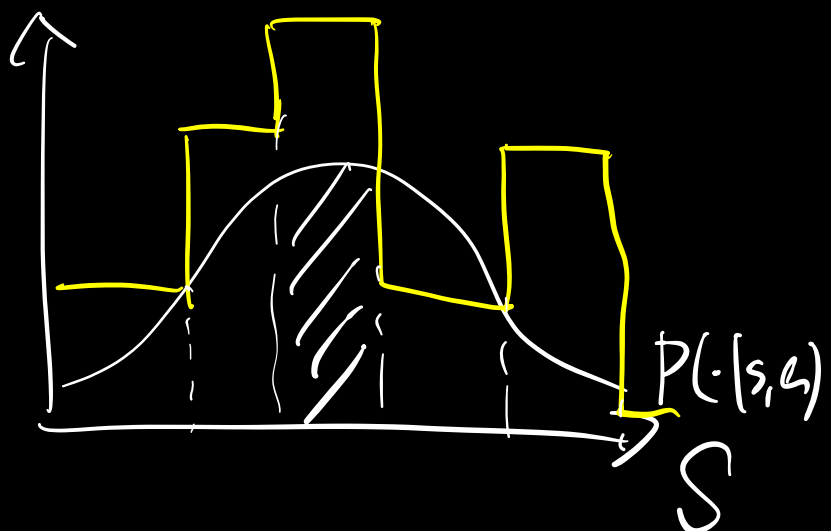
$$= R(s,a) + \gamma \, \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[[V^*_{M_\phi}]_M(s') \right]$$

$$= R(s,a) + \gamma \langle P(\cdot|s,a), \Phi^T V^*_{M_\phi} \rangle$$

$$= R_\phi(\phi(s),a) + \gamma \langle \Phi \times P(\cdot|s,a), V^*_{M_\phi} \rangle$$

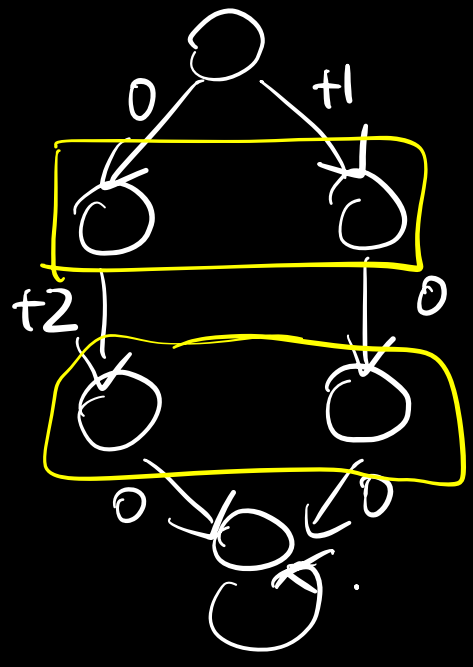
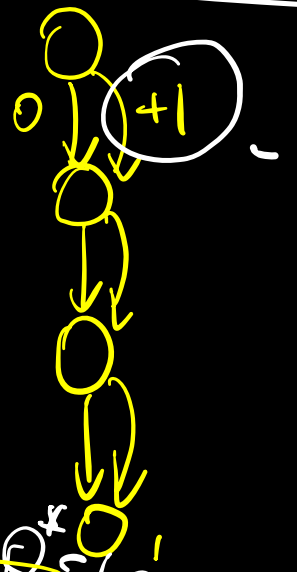
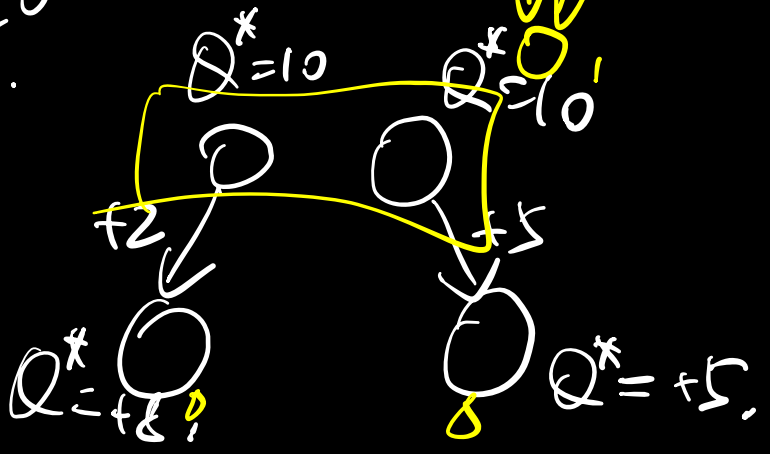
$$= Q^*_{M_\phi}(\phi(s),a) \qquad P_\phi(\cdot \mid \phi(s),a)$$

$= Q_{\mu_\phi}(s, a)$.



$\langle u, A^T v \rangle$

$= u^T A^T v$

$= (Au)^T v$

$= \langle Au, v \rangle$.