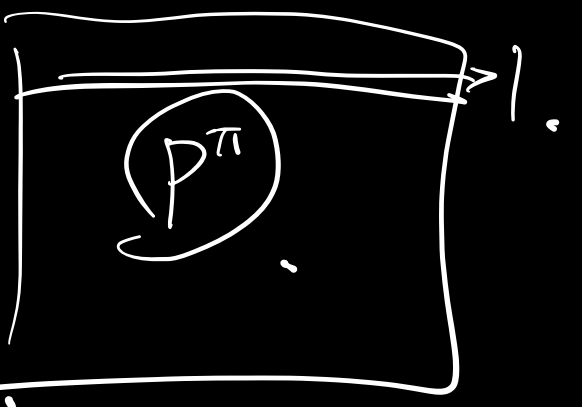
Show $I - \gamma P^\pi$ is invertible. $\gamma < 1$.

$\forall \vec{x} \neq \vec{0}$

$(I - \gamma P^\pi)\vec{x} \neq \vec{0}$.




$\|(I - \gamma P^\pi)\vec{x}\|_\infty \overset{?}{\geq} 0$.

$= \|\vec{x} - \gamma P^\pi \vec{x}\|_\infty$

$\geq \|\vec{x}\|_\infty - \gamma \|P^\pi \vec{x}\|_\infty \leq \gamma \|\vec{x}\|_\infty$.

$\geq (1-\gamma)\|\vec{x}\|_\infty > 0$.

$\|x\|_\infty = \max_{s \in S} |x(s)|$.

$\|P^\pi \vec{x}\|_\infty \longrightarrow |P^\pi x| \leq \|x\|_\infty$.

$|P^\pi x| \leq P^\pi |\vec{x}|$