

$$V^\pi(s) = \mathbb{E}_\pi \left[\sum_{t=1}^{\infty} \gamma^{t-1} r_t \mid s_1 = s \right] \quad \left| \begin{array}{l} \pi \text{ deterministic.} \\ \hline \end{array} \right.$$

$$= \mathbb{E}_\pi \left[r_1 + \sum_{t=2}^{\infty} \gamma^{t-1} r_t \mid s_1 = s \right].$$

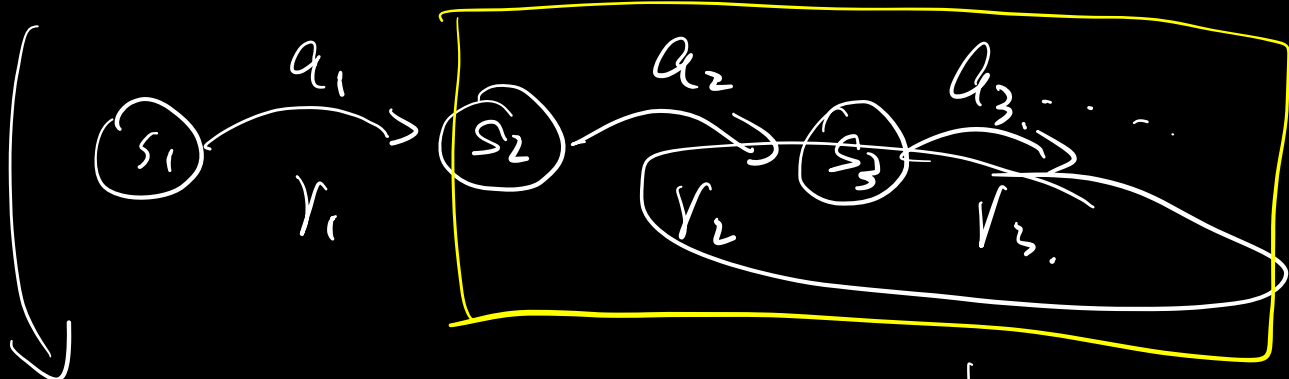
$$= \mathbb{E}_\pi [r_1 \mid s_1 = s] + \gamma \mathbb{E}_\pi \left[\sum_{t=2}^{\infty} \gamma^{t-2} r_t \mid s_1 = s \right].$$

$$= R(s, \underline{\pi(s)}) + \gamma \mathbb{E}_\pi \left[\mathbb{E}_\pi \left[\sum_{t=2}^{\infty} \gamma^{t-2} r_t \mid \begin{array}{l} s_1 = s, \\ s_2 = s' \end{array} \right] \right]$$

$f_v(s, s')$

$$= R(s, \pi(s)) + \gamma \sum_{s'} P(s' \mid s, \pi(s)) \mathbb{E}_\pi \left[\sum_{t=1}^{\infty} \gamma^{t-1} r_{t+1} \mid s_1 = s, s_2 = s' \right].$$

$$\mathbb{E}_\pi \left[\sum_{t=1}^{\infty} \gamma^{t-1} r_{t+1} \mid s_1 = s, s_2 = s' \right]$$



$$\mathbb{E}_\pi \left[\underline{r_1} + \gamma \underline{r_2} + \gamma^2 \underline{r_3} + \dots \mid \underline{s_1 = s'} \right].$$

$$= V^\pi(s').$$

$$V^\pi(s) = R(s, \pi(s)) + \gamma \sum_{s'} P(s'|s, \pi(s)) \cdot V^\pi(s').$$

$I - \gamma P^\pi$ is inv.

$$\boxed{\forall x \neq \vec{0}}$$

$$\|(I - \gamma P^\pi)x\|_\infty$$

$$\begin{aligned} [P^\pi]_{s,s'} &= P^\pi(s'|s) \\ &= P(s'|s, \pi(s)) \end{aligned}$$

$$= \|x - \gamma P^\pi x\|_\infty$$

$$\|v\|_\infty := \max_i |v_i|$$

$$\geq \|x\|_\infty - \|\gamma P^\pi x\|_\infty < \|x\|_\infty$$

$$= \|x\|_\infty - \gamma \|P^\pi x\|_\infty \geq (1-\gamma) \|x\|_\infty$$

It suffices to show: $\|P^\pi x\|_\infty \leq \|x\|_\infty$.

$$\forall s: \left| \sum_{s'} P^\pi(s'|s) x(s') \right| \leq \|x\|_\infty$$

$$\left| \mathbb{E}_{s' \sim P^\pi(\cdot|s)} [x(s')] \right| \leq \max_{s'} |x(s')|$$