Avg Bellman error: $\mathcal{E}^h(f,\pi) := \mathbb{E}_{a_{1:h-1}\sim\pi,\ a_h\sim f}[f(x_h,a_h) - r_h - \max_{a\in\mathcal{A}} f(x_{h+1},a)]$

$a_h = \arg\max f(x_h,\cdot)$

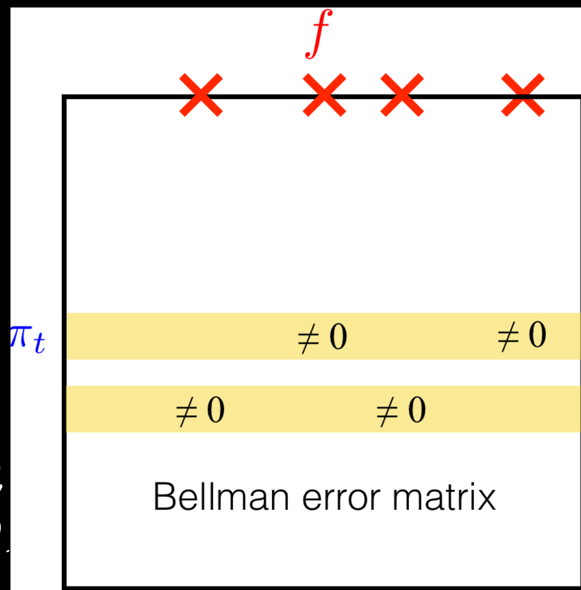
OLIVE: For $t = 1, 2, 3, \ldots,$

1. $f_t = \arg\max_{f\in\mathcal{F}_t} V_f := \max_a f(x^0, a)$

2. Collect data w/ $\pi_t = \pi_{f_t}$ & estimate $\mathcal{E}^{h_t}(f,\pi_t)$ for all f & some h_t.

$|\hat{\mathcal{E}}^{h_t}(f,\pi_t) - \mathcal{E}^{h_t}(f,\pi_t)| \le \varepsilon'$ for some h_t.

3. $\mathcal{F}_{t+1} := \{ f\in\mathcal{F}_t : |\hat{\mathcal{E}}^{h_t}(f,\pi_t)| \le \varepsilon' \}$



U P

Lemma: $\exists f\in\mathcal{F}_t,\ h_t\in[H].\ |\mathcal{E}^{h_t}(f,\pi_t)| > \frac{\varepsilon}{H}.$

Proof: let $f = f_t$.

$$\varepsilon < J(\pi^*) - J(\pi_t) \le V_{f_t} - J(\pi_t)$$

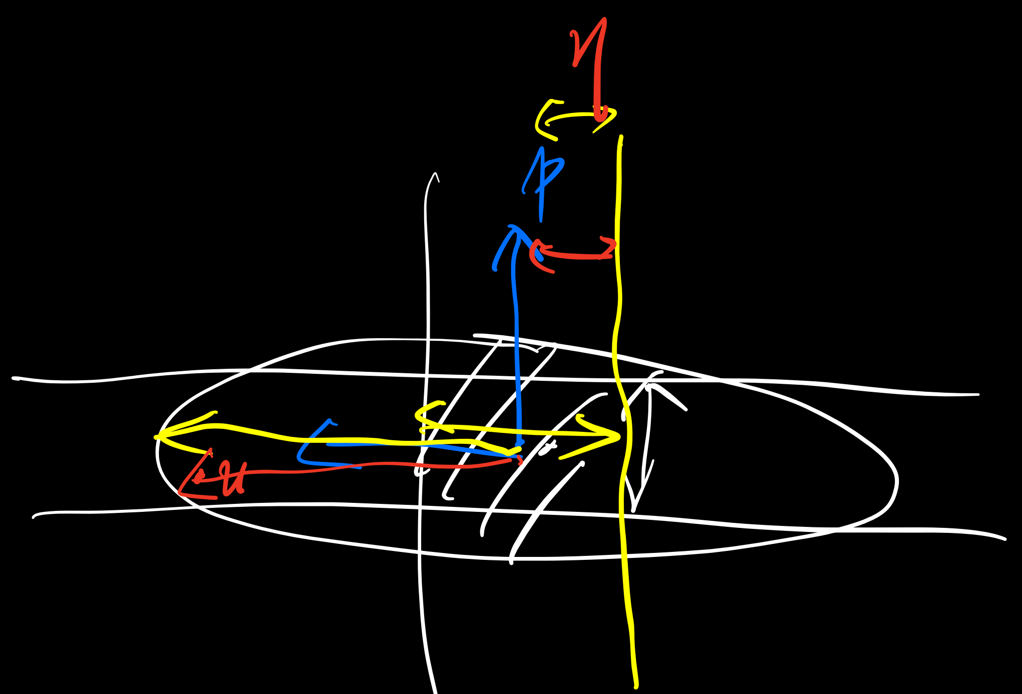
$$= \sum_{h=1}^{H} \mathcal{E}^h(f_t,\pi_t)$$

$$\Rightarrow \exists h_t.\ \mathcal{E}^{h_t}(f_t,\pi_t) > \frac{\varepsilon}{H}.$$

Set $\underline{\underline{\eta < \frac{\varepsilon}{H} / 3\sqrt{\underline{\underline{d}}}}}$

$$\frac{|A| \cdot \log \frac{|F|}{\delta}}{\eta^2} = \frac{H^2 |A| \cdot \log \frac{|F|}{\delta} \cdot d}{\varepsilon^2}$$

$$\frac{d^2 H^3 |A| \log \frac{|F|}{\delta}}{\varepsilon^2}.$$

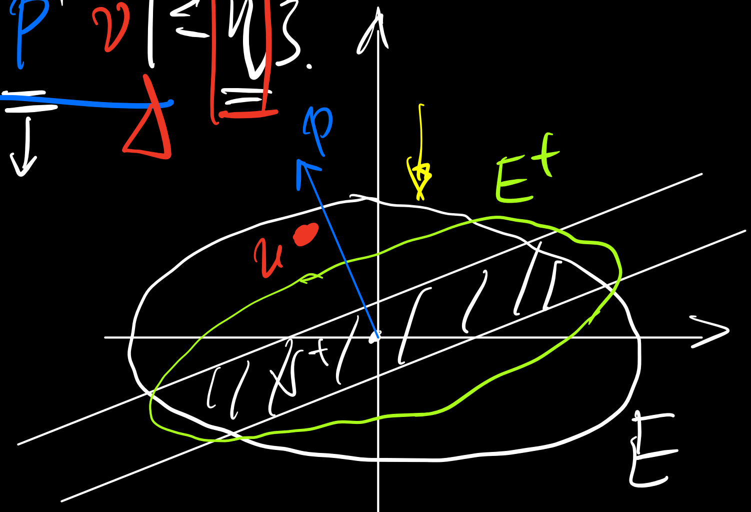


Lemma (adapted from Todd'82).

Let $E \subseteq \mathbb{R}^d$ be a centered ellipsoid.

Let $V^+ = \{ v \in E : |p^T v| \le \boxed{\eta} \}$.

for some $p \in \mathbb{R}^d$.

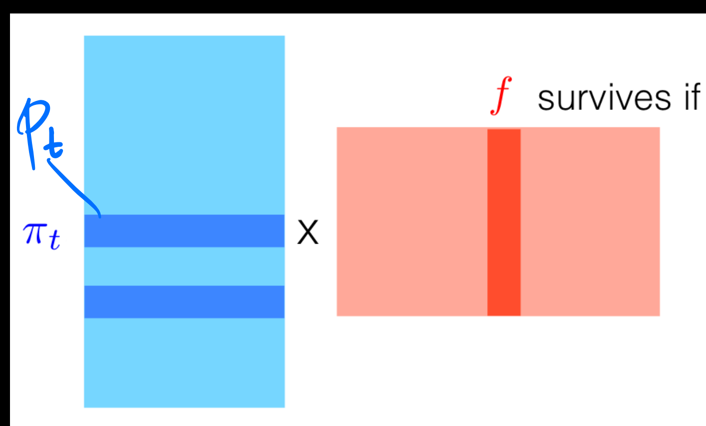


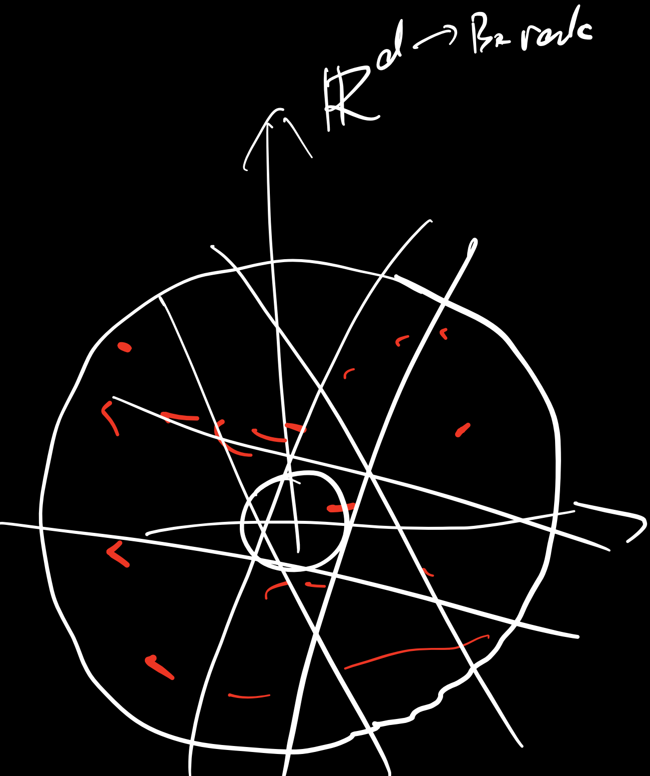
Let E^+ be the

MVEE of V^+

Then. if $\exists\, u \in E$, w/ $|u^T p| \ge 3\sqrt{d}\,\eta$.

$$\frac{\mathrm{vol}(E^+)}{\mathrm{vol}(E)} \le 0.6.$$





$$\frac{C^d}{\eta^d}$$

$$C^d \xrightarrow{0.6} \eta^d$$

$$C^d \cdot 0.6^{\boxed{T}} \leq \eta^d.$$

$$0.6^T \leq \left(\frac{\eta}{C}\right)^d$$

$$T \geq \log_{\frac{5}{3}} \left(\frac{C}{\eta}\right)^d$$

$$= d \log_{\frac{5}{3}} \left(\frac{C}{\eta}\right).$$