Avg Bellman error: $\mathcal{E}^h(f,\pi) := \mathbb{E}_{\substack{a_{1:h-1}\sim\pi \\ a_h\sim f}}\left[f(x_h,a_h) - r_h - \max_{a\in\mathcal{A}} f(x_{h+1},a)\right]$

$a_h = \arg\max f(x_h,\cdot)$

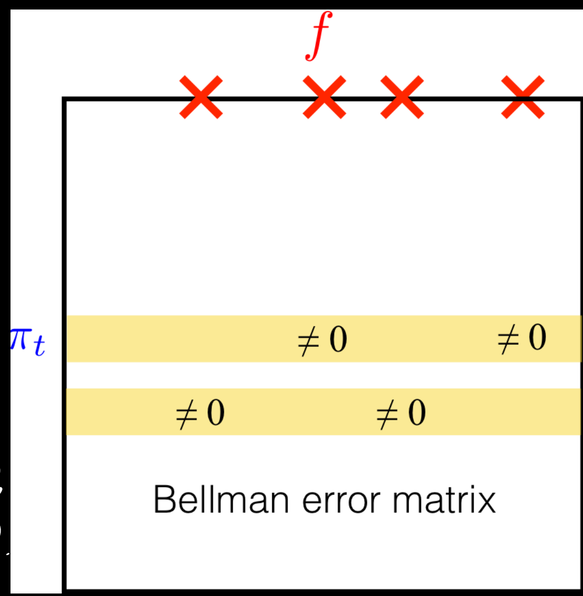
OLIVE: For $t = 1, 2, 3, \ldots,$

1. $f_t = \arg\max_{f\in\mathcal{F}_t} V_f := \max_a f(x^0,a)$

2. Collect data w/ $\pi_t = \pi_{f_t}$ & estimate $\mathcal{E}^{h_t}(f,\pi_t)$ for all f. & some h_t.

$$|\hat{\mathcal{E}}^{h_t}(f,\pi_t) - \mathcal{E}^{h_t}(f,\pi_t)| \le \varepsilon'$$

for some h_t.



3. $\mathcal{F}_{t+1} := \{f\in\mathcal{F}_t : |\hat{\mathcal{E}}^{h_t}(f,\pi_t)| \le \varepsilon'\}$

$$\Rightarrow \forall f\in\mathcal{F}_{t+1} \quad |\mathcal{E}^{h_t}(f,\pi_t)| \le 2\varepsilon'.$$

Lemma: $\exists f\in\mathcal{F}_t, \quad |\mathcal{E}^{h_t}(f,\pi_t)| > \frac{\varepsilon}{H}.$

Proof: $\frac{\varepsilon}{H} < V_{f_t} - J(\pi_t) = f_t(x^0,\pi_t) - J(\pi_t)$

$$= \sum_{h=1}^{H} \mathbb{E}_{\substack{x_h\sim d_h^{\pi_t} \\ a_h\sim\pi_{f_t}}}\left[f_t(x_h,a_h) - r_h - \max_a f_t(x_{h+1},a)\right].$$

$$= \sum_{h=1}^{H} \mathcal{E}^h(f_t,\pi_t)$$

$\exists\, h,\ \varepsilon^h(f_t, \pi_t) > \frac{\varepsilon}{H}.$

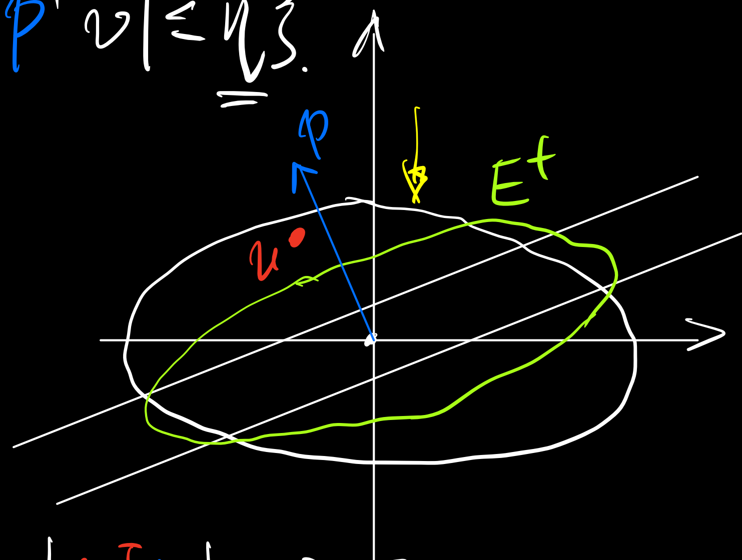
$\hookrightarrow h_t.$

Lemma (adapted from Todd'82).

Let $E \subseteq \mathbb{R}^d$ be a centered ellipsoid.

Let $V^+ = \{ v \in E : |p^\top v| \le \eta \}$.

for some $p \in \mathbb{R}^d$.



Let E^+ be the MVEE of V^+

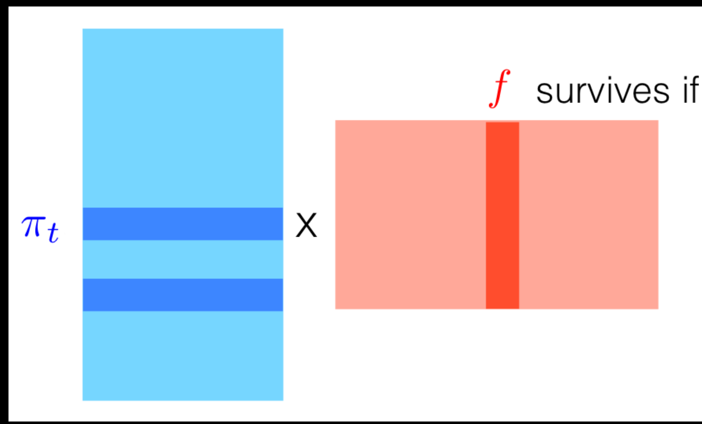
Then. if $\exists\, u \in E$, w/ $|u^\top p| \ge 3\sqrt{d}\,\eta$.

$$\frac{\mathrm{vol}(E^+)}{\mathrm{vol}(E)} \le 0.6.$$

$x \in \mathbb{R}.\ \|x\|_2 \le r.$

$x^\top E x \le r^2.$

$E \in \mathbb{R}^{d\times d}.$



$\eta \longleftrightarrow 2\varepsilon'$.

$|p^T u| \longleftrightarrow \dfrac{\varepsilon}{H}$.

$d \longleftrightarrow B$.

$$\frac{\varepsilon}{H} \geq O(\sqrt{B}\cdot\varepsilon').$$

$\|$ red vector $\| \leq C$.

so. $\mathrm{vol}(E_0) \sim C^B$.

$\mathrm{vol}(E_{final}) \sim (\varepsilon')^B$.

$$\left(\frac{C}{\varepsilon'}\right)^B \gtrsim \frac{\mathrm{Vol}(E_0)}{\mathrm{Vol}(E_{final})} \geq \left(\frac{5}{3}\right)^{\#iter}$$

$$\Rightarrow \quad \#iter \leq \log_{\frac{5}{3}} \left(\frac{C}{\varepsilon'}\right)^B$$

$$= B \log_{\frac{5}{3}}\left(\frac{C}{\varepsilon'}\right).$$

$$\simeq O(B\log B)$$