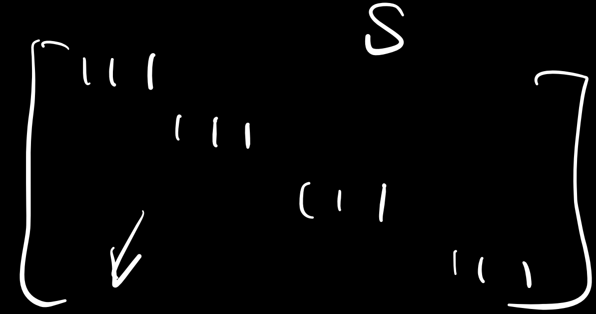
$\Phi(x,s) = \mathbb{I}[\phi(s) = x]$.

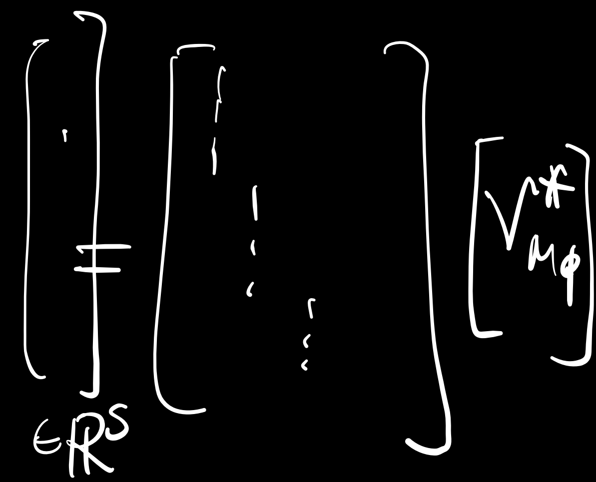
$S_\phi$ 

① lift..

$V^*_{M_\phi} \in \mathbb{R}^{|S_\phi|}$.

$s \mapsto V^*_{M_\phi}(\phi(s))$. $= V^*_M(s)$ ?

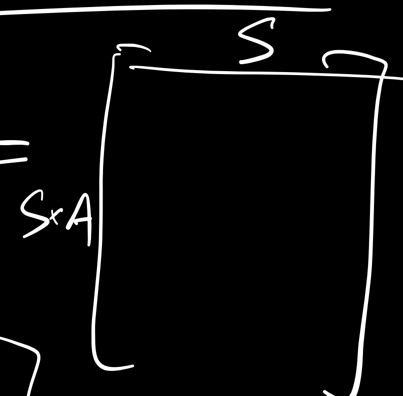
$\Phi^\top V^*_{M_\phi}$

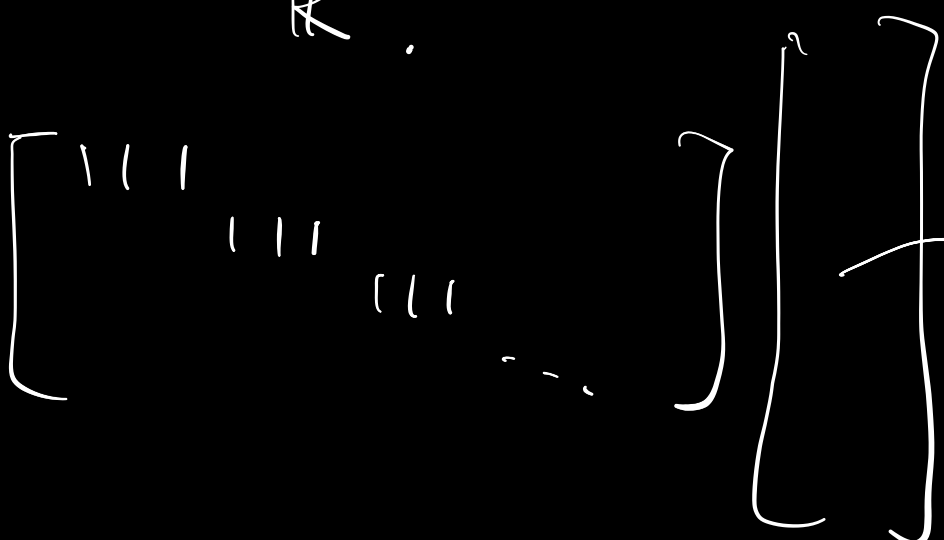


② $P(x' \mid s,a) = \sum_{s' \in \phi^{-1}(x)} P(s' \mid s,a)$.

$\Phi P(\cdot \mid s,a)$

$\in \mathbb{R}^S$.

$P = $ 



$\to P(\cdot \mid s,a) \in \mathbb{R}^S$.

$\forall s^{(1)}, s^{(2)}, \quad R(s^{(1)}, a) = R(s^{(2)}, a)$

$\Phi \times P(\cdot \mid s^{(1)}, a) = \Phi \times P(\cdot \mid s^{(2)}, a)$.

---

Statement : $Q^*_{M_\phi}(\phi(s), a) = Q^*_M(s, a)$.

Suffices : $[Q^*_{M_\phi}]_M \in \mathbb{R}^{|S \times A|}$:

$[Q^*_{M_\phi}]_M(s, a) = \mathcal{T}_M [Q^*_{M_\phi}]_M(s, a)$

$R(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} [[V^*_{M_\phi}]_M(s')]$.

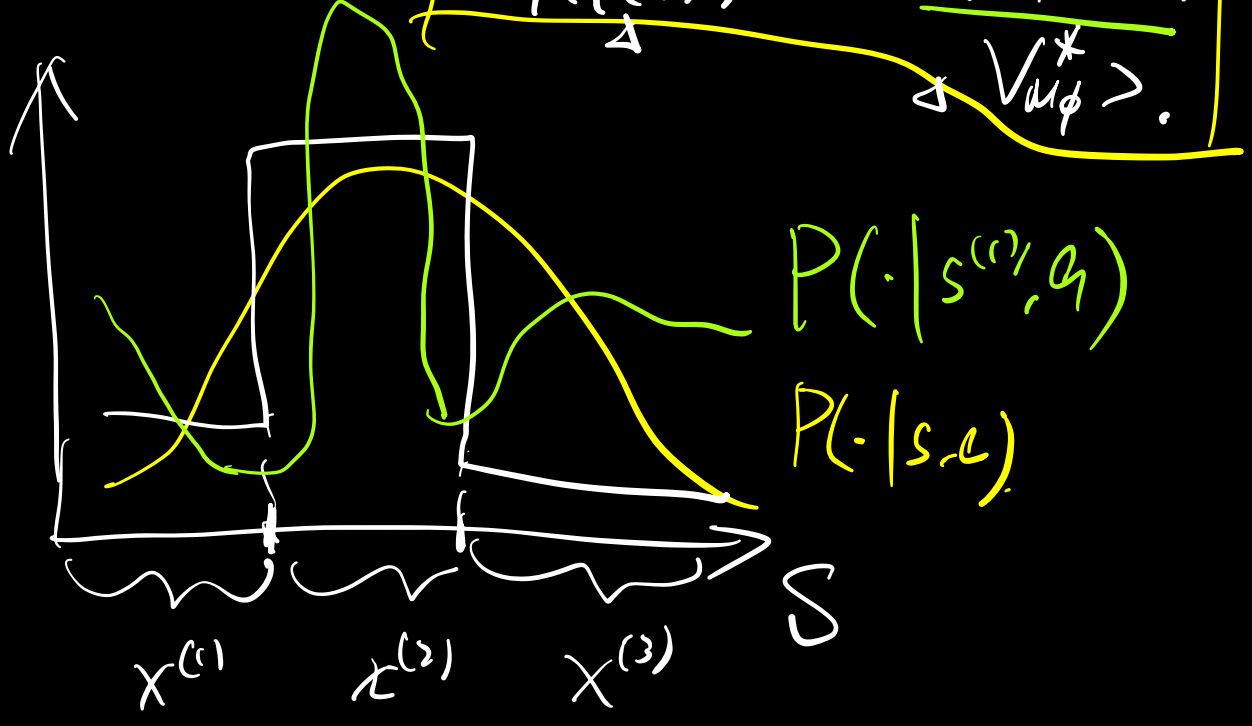
$= R(s, a) + \gamma \langle P(\cdot \mid s, a), \Phi^T V^*_{M_\phi} \rangle$

$= R_\phi(s, a) + \gamma \langle \Phi P(\cdot \mid s, a), V^*_{M_\phi} \rangle$

$\langle u, Av \rangle = u^T A v = (A^T u)^T v = \langle A^T u, v \rangle$

$= [Q^*_{M_\phi}]_M(s, a) = Q^*_{M_\phi}(\phi(s), a)$

$= R_\phi(\phi(s), a) + \gamma \langle P_\phi(\cdot \mid \phi(s), a),$


$P(\cdot|s^{(1)},q)$
$P(\cdot|s,c)$
$s$
$x^{(1)}$
$x^{(2)}$
$x^{(3)}$