

Importance Sampling

$$(x, a, r) \sim \underline{g}$$

Data:

$$\begin{aligned} x &\sim d_0, \\ a &\sim \underline{\pi_b}(\cdot | x), \\ r &\sim R(\cdot | x, a). \end{aligned}$$

$$\begin{aligned} x &\sim d_0, \\ a &\sim \pi(\cdot | x), \\ r &\sim R(\cdot | x, a). \end{aligned}$$

Estimate goal: $\mathbb{E}[r | \underline{\pi}]$

$$(x, a, r) \sim p$$

$$= \mathbb{E}_{(x, a, r) \sim p} [r]$$

$$= \mathbb{E}_{(x, a, r) \sim g} \left[\frac{p(x, a, r)}{g(x, a, r)} \cdot r \right]$$

$$p(x, a, r) = p(x) \cdot \underline{p(a|x)} \cdot \underline{p(r|x, a)}$$

$$= d_0(x) \cdot \pi(a|x) \cdot R(r|x, a)$$

$$g(x, a, r) = \cancel{d_0(x)} \cdot \pi_b(a|x) \cdot \cancel{R(r|x, a)}$$

$$\frac{\mathcal{P}(x, a, r)}{\mathcal{Q}(x, a, r)} = \frac{\pi(a|x)}{\pi_b(a|x)}$$

$$(x, a, r) \rightarrow \frac{\pi(a|x)}{\pi_b(a|x)} \cdot r.$$

